The Development of a Model for Estimating a Velocity Function for the Money Supply: A Tool for Policymakers

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This paper develops a model for the velocity of the M2 money stock that simultaneously estimates the functional form and the parameters of the velocity equation. The study is based on the data since 1991, a period characterized by an abrupt shift in the velocity level. The results show that a Box-Cox model may be more appropriate than a logarithmic model for specifying a velocity function. The accepted model and the estimated coefficients should have useful implications for the policymakers.

I. Introduction

A stable and predictable velocity of money has important implications for macroeconomic policy because the velocity is a link between the monetary aggregate and nominal GDP. From 1970 through the mid-1980s the Federal Reserve Board found M1 (currency plus demand deposits, traveler's checks and other checkable deposits) to be a useful gauge of economic and financial conditions and employed it as an intermediate target for achieving macroeconomic objectives. In recent years, however, M1 has been found to be less useful since its velocity could not be explained or estimated with reliability. The Fed is now considering M2 (M1 plus saving deposits and money market mutual fund balances) as a potential tool for attaining desirable levels of GDP, controlling inflation and pursuing other macroeconomic objectives. Consequently, the behavior of the M2 velocity is becoming the subject of much research (see for example Carlson et al (2000), Duca (2000)). Feldstein and Stock (1994) investigated the relationship between M2 and nominal GDP and found it to be stable. Since the mid-1990s, however, most models predicted the quantity of M2 to be less than actual amount and correspondingly the velocity to be higher than its actual value. Some authors have labeled this as the case of missing money since for the given levels of GDP the level of M2 should have been higher. Duca (2000) believes that the "missing M2" problem may be resolved when the effect of the shift in bond mutual fund costs on the quantity of money is taken into account. Carlson, et al (2000) argued that the recent instability in the magnitude of the M2 velocity was characterized by a permanent upward shift, which began around 1990 and was largely over 1994. They hypothesized that during this period, households permanently reallocated a part of their assets from time deposits to mutual funds and argued that this substitution could be explained by an appropriately measured opportunity cost. A number of other papers have analyzed the characteristics of M2 velocity in recent years (e.g. Feinman and Porter (1992), Estrella and Mishkin (1997), Orphanides and Porter (1998)), but no consensus has been reached regarding the appropriate method of specifying the relationship between the M2 velocity and its explanatory variables. It should therefore be useful to further explore this relationship before a monetary aggregate such as M2, could be used as a policy alternative by the Fed.

This paper reconsiders the velocity function derived from a standard money demand equation. A Box-Cox transformation of variables is used to specify the functional form of the income velocity of money. In a time-series regression, seemingly autoregressive residuals may be due to an incorrect functional specification or autocorrelation or both. The Box-Cox model of Savin and White (1978) will, therefore, be used to simultaneously estimate the functional form of the equation and test for autocorrelation. A likelihood ratio test is employed to compare alternative specifications of the velocity function.

The equation specifying the velocity function is described in section 2. The data and empirical results are presented and analyzed in section 3. Concluding remarks are made in section 4.

II. The Model

The demand for real balances is usually specified as

$$\frac{M_t}{P_t} = m_t = f(Y_t, R_t, m_{t-1}) \tag{1}$$

where M_t is the nominal stock of money (M2), P_t is the price level, Y_t is the real income, R_t is the difference between the representative interest rate (yield on 30 year Aaa rated corporate bonds) and the own return on M2. The velocity of money is defined as

$$V_t = \frac{Y_t P_t}{M_t} = \frac{Y_t}{m_t} \tag{2}$$

Combining (1) and (2) yields

$$V_t = \frac{Y_t}{f(Y_t, R_t, m_{t-1})}$$

or

$$V_t = g(Y_t, R_t, m_{t-1}) \tag{3}$$

A Box-Cox transformation of variables is used to estimate the functional form and coefficients of the equation specifying the relationship shown in (3).

$$V_t^* = \alpha + \beta_1 Y_t^* + \beta_2 R_t^* + \beta_3 m_{t-1}^* + \varepsilon_t \tag{4}$$

where $V_t^\lambda = \frac{V_t^\lambda - 1}{\lambda}$ and $V_t^* = V_t^{(\lambda)} - \rho V_{t-1}^{(\lambda)}$

ρ is the first-order autoregressive parameter and λ is the Box-Cox transformation parameter. ε_t is assumed to be normally and independently distributed with zero mean and constant variance. The asterisks on the right hand side variables in equation (4) represent transformations of the variables for the Box-Cox autoregressive model, similar to that shown for V^*. Equation (4) reduces to a logarithmic function when $\lambda = 0$ and to a linear equation when $\lambda = 1$.

The concentrated log-likelihood function for equation (4) except for a constant can be expressed as

$$L(\lambda,\rho) = -\frac{N}{2}\ln\lambda\hat{\sigma}^2(\lambda,\rho) + \frac{1}{2}\ln(1-\rho^2) + (\lambda-1)\sum_{t=1}^{N}\ln V_t \quad (5)$$

where N is the sample size and $\hat{\sigma}^2$ is the estimated residual variance. The maximized value of $L(\lambda,\rho)$ can be obtained by regressing V^*_t on Y^*_t, R^*_t, and m^*_{t-1} over different values of λ and ρ. The test statistic is given by

$$J(\lambda,\rho) = 2[L(\phi) - L(\phi_r)] > \chi^2_\alpha(v)$$

where ϕ is the parameter space under the maintained hypothesis, ϕ_r is the space ϕ restricted by the null hypothesis, v is the degrees of freedom. The likelihood ratio test of the null hypothesis with restricted parameters versus the alternative hypothesis without restrictions on the parameters can determine whether a specific functional form is appropriate. Since the double-log form is a standard specification of the velocity function, the Box-Cox model in equation (4) will be tested against the null hypothesis that the double-log model is the null hypothesis that the double-log model is the appropriate specification of the velocity function.

Ceteris paribus, the velocity of money varies directly with the level of real income. The coefficient of real income in a money demand function is usually less than 1, which also suggests that the coefficient of income (β_1) in equation (4) should be positive. Since the velocity is inversely related to demand for money, the sign of β_2 is expected to positive and the sign of β_3 is expected to be negative.

III. Empirical Results

Quarterly data from 1991:Q1 to 2004:Q4 were used to estimate the parameters of velocity function. Real income is measured by real GDP, M2 is the money stock, and the GDP deflator is the price level. The interest rate used is the 30-year Aaa corporate bond yield adjusted for the own return on M2. This provides a more accurate measure of the opportunity cost of holding money. The long rate is used because it represents a weighted average of expected future short rates. Another argument for the long rate is that capital investments, which are substitutes for money, are based on long rates[1]. The data has been obtained from the Federal Reserve Bank of St Louis. All the variables in the time series were found to be non-stationary in level but were stationary in first difference, so the model specified by equation (4), depicting a long-term relationship is still valid.

The regression results are presented in Table 1. The results show that the velocity is directly related to interest rates. This is expected since the demand for money is likely to drop with a rise in interest rates, leading to a higher velocity. The coefficients of the other variables also have the expected signs and are significant at the 1% level. The estimated value of ρ is also significant at the 5% level for the double-log model, suggesting that it is appropriate to consider autocorrelation and functional form simultaneously. The value of the adjusted R^2 indicates that over 99% of the movements of the velocity may be explained by the standard variables using the functional form shown in equation (4). The value of the log-likelihood function $L(\lambda,\rho)$ is found to be maximized in the Box-Cox model. A chi-square test of the log-likelihood functions shows that the test statistic is equal to 10.621. Since the p-value for the test statistic (with one degree of freedom) is 0.001, we can reject the double-log model in favor of the Box-Cox model. The relevant elasticities at means are also reported in Table 1.

Table 2 compares the predictions from the Box-Cox estimation with that of the Logarithmic model. The results show that the Box-Cox model is more reliable in tracking the velocity from 1991 to 2004.

IV. Conclusion

The velocity of money is important for macroeconomic policy since it serves as a link between the monetary aggregate and nominal GDP. The instability and abrupt rise in the velocity of money since the beginning of 1991 has therefore prompted extensive research efforts to explain the apparent break in its the trend. This paper investigated a model, which simultaneously estimates the functional form and tests for autocorrelation of the velocity, based on recent data. Our major findings are: (1) a theoretical velocity function derived from a money demand function is useful for estimating its empirical counterpart, (2) a conventional double-log form chosen a priori for the velocity function is rejected, and (3) a general functional form represented by the Box-Cox model is useful for estimating the velocity equation. The results show that reliable estimates of M2 velocity may be obtained for the 1990s, which was characterized by an abrupt shift from its historic trend. This should have useful implications for macroeconomic policy.

Footnote

1 See Poole (1988) for a similar justification

Table 1. Estimated Velocity Regression

Variable	Box-Cox Model		Double Log Model	
	Coefficient Estimates	Elasticity at the means	Coefficient Estimates	Elasticity at the means
Constant	0.806 (36.65)	0.670	0.663 (58.17)	0.663
Y_t	0.102 (86.24)	0.977	0.0001 (50.51)	1.018
R_t	0.022 (6.33)	0.028	0.009 (5.53)	0.040
m_{t-1}	-0.128 (-83.52)	-1.019	-0.0003 (-49.19)	-1.035
λ	-0.27		0	
ρ	0		0.31 (2.26)	
DW	2.05		2.03	
$L(\lambda,\rho)$	166.961		156.340	
$\bar{R}^2$	0.995		0.992	

Table 2. Comparison of Forecasting Errors

	Box-Cox Model	Double Log Model
Mean Square Error	0.000055	0.000094
Mean Absolute Error	0.059690	0.007818
Root Mean Square Error	0.007422	0.009672
Mean Square Percentage Error	0.142210	0.242350

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